October 7, 2004; Due October 19, 2004.

Note: this assignment is shorter than usual. I hope to return it, graded, before the midterm, hence the shorter deadline.

Math 5652: Homework set #3: Markov Chains (Fall 2004)

1) We have proved in class that if a MC is irreducible and a-periodic, and possesses a stationary distribution π, then $P^n(x,y)$ converges to $\pi(y)$. Prove that the same conclusion holds true if the assumption of irreducibility is dropped and instead one assumes that the set of recurrent states, denoted R, is closed, irreducible, and a-periodic, and that there exists an $\alpha > 0$ such that for any transient x, $\sum_{y \in R} P(x,y) \geq \alpha$.

2) Let X_n be a homogeneous Markov chain, having as state space the integers, with transition probability matrix $P(x,y) = p1_{y=x+1} + (1-p)1_{y=x-1}$. Let $\tau = \min\{n : |X_n| = 1000\}$. Compute a) $P_0(X_\tau = 1000)$ b) $E_0\tau$.

3) Construct an example of a homogeneous Markov chain with state space the integers, such that all states are transient.

4) Construct an example of a homogeneous Markov chain with state space the integers, such that $P^n(x,y)$ does not converge for any x, y but $P^{2n}(x,y)$ does.

5) A martingale is a stochastic process $\{X_n\}$ satisfying $E[X_{n+1}|X_1, ..., X_n] = X_n$. Let P be the transition matrix of a homogeneous Markov chain $\{Y_n\}$, and for any function f, define $Pf(x) = \sum_y P(x,y)f(y)$. Prove that $X_n = f(Y_n) - Pf(Y_{n-1})$ is a martingale.

Assignment rules: Submitted work must be your own. You may, and in fact are encouraged to, collaborate on an assignment, provided that no more than 2 people are collaborating. In such case, you are requested to note the names of your collaborators on your submission. If collaboration is significant (more than two questions), you are requested to jointly submit your assignment.