November 2, 2004. Due November 16,2004

Math 5652: Homework set #5: Continuous time Markov chains (Fall 2004)

1) A random telegraph noise is a time homogeneous Markov process X_t with states $\{0, 1\}$ and rates $\lambda_{0,1} = \lambda_{1,0}$. Given that $X_0 = 0$, compute $P(X_t = 1)$. Find the stationary distribution for X_t.

2) Let Q be the rate matrix for a finite state continuous time Markov process X_t. Suppose one is given that Q is a symmetric matrix. Find the stationary distribution of the process.

3) Let $\{X_t\}$ be a finite state, time homogeneous Markov chain, and let $\{X_n\}$ be the discrete time process obtained by sampling it at integer times. Prove that $\{X_n\}$ is a Markov chain, compute its transition matrix in terms of the rate matrix for X_t, and find the stationary distribution of $\{X_n\}$ in terms of that for $\{X_t\}$.

4) A service station has two states: broken and active. The time between failures is exponential μ, and the time to repair is exponential λ. All failure times and repair times are independent. Evaluate

$$\lim_{t\to\infty} \frac{\text{Total time station is active before time } t}{t}$$

5) Exercise 8.24 from book (chapter 4).

6) Exercise 8.8 from book (chapter 4).

7) Let τ_i be i.i.d., positive random variables, with distribution function F. Define $T_k = \sum_{i=1}^{k} \tau_i$ and $N_t = \max\{k : T_k \leq t\}$. Find necessary and sufficient conditions on F that ensure that $\{N_t\}$ is a time homogeneous Markov process.

This problem may look familiar, but it is much easier than the impossible problem I gave in HW4!

Assignment rules: Submitted work must be your own. You may, and in fact are encouraged to, collaborate on an assignment, provided that no more than 2 people are collaborating. In such case, you are requested to note the names of your collaborators on your submission. If collaboration is significant (more than two questions), you are requested to jointly submit your assignment.