November 16, 2004. Due November 30,2004

# Math 5652: Homework set #6: Brownian Motion

(Fall 2004)

1) Recall that with $B_t$ denoting Brownian motion, the Orenstein-Uhlenbeck process $\{X_t\}$ is the process $X_t = e^{-t} B_{e^{2t}}$. Prove that $X_t$ is a homegeneous Markov process.
2) Exercise 6.21 in book (chapter 6).
3) Exercise 6.16 in book (chapter 6).
4) Exercise 6.22 in book (chapter 6).
5) For $x \in R$, let $W^x(t) = x + B_t$ where $\{B_t\}$ is a Brownian motion. Compute $v(x, t, p) = E(W_x(t)^p)$ for integer $p$ and prove that

$$\frac{d}{dt} v(x, t, p) = \frac{1}{2} \frac{d^2}{dx^2} v(x, t, p) \, .$$

6) Let $B_t$ be a Brownian motion, and set $\tau_0 = 0$, $\tau_i = \min\{t > \tau_{i-1} : |X_t| = 1, X_t \neq X_{\tau_{i-1}}\}$. Prove that $Y_i = X_{\tau_{i+1}}$, $i \geq 0$, is a homogeneous, stationary Markov process.
**Assignment rules:** Submitted work must be your own. You may, and in fact are encouraged to, collaborate on an assignment, provided that no more than 2 people are collaborating. In such case, you are requested to note the names of your collaborators on your submission. If collaboration is significant (more than two questions), you are requested to jointly submit your assignment.