Course description: Large deviations and applications

Spring semester 2003, Math 8660.

The simplest example of a large deviations computation is the following. Let $\{X_i\}$ be a sequence of i.i.d., Bernoulli variables of parameter 1/2, and set $S_n = \sum_{i=1}^n X_i$. Then, for $x \in [0,1]$,

$$\lim_{\delta \to 0} \lim_{n \to \infty} n^{-1} \log P(S_n/n \in (x-\delta, x+\delta)) = -I(x)\,, \qquad (1)$$

where $I(x) = x \log x + (1-x)\log(1-x) + \log 2$. The interest in (1) is of course when $x \neq 1/2$.

Large deviations theory is a bag of tools to prove statements of this form. It has many analogies with the theory of weak convergence of probability measures. Applications are diverse and range from statistical mechanics to information theory to statistics to sequence (=DNA) analysis to communication and radar systems.

Depending on the interests of the audience, I will cover a subset of the following:

- LD in finite dimensional state spaces: The method of types, theorems of Cramer and Gartner-Ellis, concentration inequalities for product measures.
- Applications: sequence matching, coding theory, statistics, random matrices.
- General statement of LD principle.
- General techniques: contraction and inverse contraction, Varadhan's lemma, convexity and LD, projective limits, sub additivity.
- LD for empirical measures: Sanov's theorem, Markov chains, hypermixing.
- LD for sample paths: Random walks, Brownian motion, diffusions.
- Applications: exit time control, tracking.

If you understand the statement (1) above and know some analysis (topological spaces, compactness, weak convergence) and probability with measure theory, you probably have the required prerequisites for the course. For more details, please contact Ofer Zeitouni at zeitouni@math.umn.edu

Text: Large deviations techniques and applications, A. Dembo and O. Zeitouni, 2-nd. edition, Springer (1998).