Twisted Gelfand pairs from reflection groups

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Outline

1. An eigenvalue mystery...
   - A matrix indexed by permutations
   - A matrix indexed by a reflection group
2. Some ideas
   - Idea 1: Representations
   - Idea 2: Flipping a factorization
   - Idea 3: A twisted Gelfand pair
3. Mystery solved!

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A mystery haunted our fair city...

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Consider the matrix A whose rows and columns are indexed by **permutations** σ in $\mathfrak{S}_n$,
with (σ, τ)-entry the number of pairs $i < j$ that appear in σ, τ in the **same order**.

That is, $A_{\sigma,\tau}$ counts **noninversions** of $\sigma \circ \tau^{-1}$.

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E.g., for n=3, the matrix A is

	$(1,2,3)$	$(1,3,2)$	$(2,1,3)$	$(2,3,1)$	$(3,1,2)$	$(3,2,1)$
$(1,2,3)$	3	2	2	1	1	0
$(1,3,2)$	2	3	1	0	2	1
$(2,1,3)$	2	1	3	2	0	1
$(2,3,1)$	1	0	2	3	1	2
$(3,1,2)$	1	2	0	1	3	2
$(3,2,1)$	0	1	1	2	2	3

Easy to see that

$$A_{\sigma,\tau} = A_{\tau,\sigma}$$

and hence A will have eigenvalues in **R**.

We'll see in a bit that it can be factored

$$A = \pi \circ \pi^t$$

so it even has **nonnegative** eigenvalues.

MYSTERY.
Why does A seem to have **all eigenvalues in Z**?

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Furthermore, empirically it has only **four** eigenspaces:

$$\det(tI - A) = (t-0)^{n!-1-\binom{n}{2}} \times \left(t - \frac{n!\binom{n}{2}}{2}\right)^{1} \times \left(t - \frac{(n+1)!}{6}\right)^{n-1} \times \left(t - \frac{n!}{6}\right)^{\binom{n-1}{2}}$$

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More to the mystery

The matrix A represents multiplication on the **right** by

$$A := \sum_{\sigma \in \mathfrak{S}_n} \#\{\text{ noninversions of } \sigma\} \cdot \sigma$$

as a linear operator on the group algebra $\mathbf{R}\mathfrak{S}_n$:

$$\mathbf{R}\mathfrak{S}_n \xrightarrow{(-)\cdot A} \mathbf{R}\mathfrak{S}_n.$$

It commutes with the **left**-regular action of $\mathbf{R}\mathfrak{S}_n$ on itself, so its **eigenspaces** are $\mathfrak{S}_n$-**representations.**

It also happens that A commutes with an extra $\mathbf{Z}/2\mathbf{Z}$-action coming from right-multiplication in $\mathbf{R}\mathfrak{S}_n$ by the **longest element**

$$w_0 = \begin{pmatrix} 1 & 2 & \cdots & n-1 & n \\ n & n-1 & \cdots & 2 & 1 \end{pmatrix}.$$

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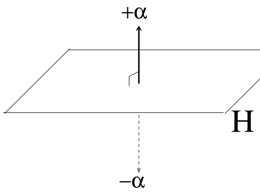
In fact, these operators arose at the intersection of **three families** that we conjectured had integer spectra. Two families we understood pretty well.

One family starts with a finite real **reflection group** W, and a choice of **positive root** normals $\{+\alpha\}$ for its collection of **reflecting hyperplanes** $\{H\}$.



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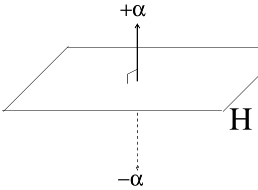

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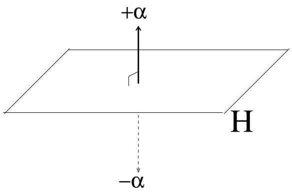

Say H is a **noninversion** for w in W if w sends the **positive** root $+\alpha$ normal to H to another **positive** root $+\beta$.

Now choose a particular reflecting hyperplane H.
Let $\mathcal{O}$ be the W-orbit of hyperplanes containing H.
Define an element A in the group algebra $\mathbf{R}W$ by

$$A := \sum_{w \in W} \# \left\{ \begin{array}{c} H' \in \mathcal{O} \text{ which are} \\ \text{noninversions for } w \end{array} \right\} \cdot w$$

Consider the eigenvalues of the linear operator

$$\mathbf{R}W \xrightarrow{(-)\cdot A} \mathbf{R}W.$$

Its eigenspaces are again $W \times \mathbf{Z}/2\mathbf{Z}$-representations.

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Then our original mystery for $W = \mathfrak{S}_n$ seemed to generalize as follows.

THEOREM.
For **Weyl** (= **crystallographic** finite reflection) groups W, and any choice of a W-orbit $\mathcal{O}$ of hyperplanes, the operator

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has all its eigenvalues in $\mathbf{Z}$.

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A recent development

We also made an empirically-based finer conjecture, independently proven recently (2011) by P. Renteln:

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For W **simply-laced**, i.e. types $A_\ell, D_\ell, E_6, E_7, E_8$, of **rank** ℓ, with N **hyperplanes**, and **Coxeter number** h, the operator A on $\mathbf{R}W$ has

$$\det(tI - A) = (t-0)^{|W|-1-N} \times \left(t - \frac{|W|N}{2}\right)^1 \times \left(t - \frac{|W|(h+1)}{6}\right)^\ell \times \left(t - \frac{|W|}{6}\right)^{N-\ell}$$

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Outline

An eigenvalue integrality principle

PROPOSITION:
Think of a matrix A in $\mathbf{Z}^{N \times N}$ as an operator $\mathbf{R}^N \xrightarrow{A} \mathbf{R}^N$.
If A commutes with the action of a finite group W on $\mathbf{R}^N$, decomposing $\mathbf{R}^N$ into W-**irreducibles**

- all realizable over $\mathbf{Q}$,
- with **no multiplicities**

then A has all its eigenvalues in $\mathbf{Z}$.

PROOF (sketch): The above assumptions, together with **Schur's lemma**, imply the eigenvalues of A lie in $\mathbf{Q}$.

But the eigenvalues are also roots of the monic polynomial $\det(tI - A)$ in $\mathbf{Z}[t]$.
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Let $\Phi_{\mathcal{O}}$ be the union of all roots $\{+\alpha, -\alpha\}$ normal to hyperplanes in the W-orbit $\mathcal{O}$.

Then it turns out $A = \pi^t \circ \pi$ where

$$\mathbf{R}W \xrightarrow{\pi} \mathbf{R}^{\Phi_{\mathcal{O}}}$$

is defined by

$$\pi_{e_w, e_\alpha} = \begin{cases} 1 & \text{if } w(\alpha) \text{ is a positive root,} \\ 0 & \text{otherwise.} \end{cases}.$$

In fact, the map π is even $W \times \mathbf{Z}/2\mathbf{Z}$-equivariant if one lets $\mathbf{Z}/2\mathbf{Z}$ act on $\mathbf{R}^{\Phi_{\mathcal{O}}}$ swapping the basis elements $e_{+\alpha} \leftrightarrow e_{-\alpha}$.

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Rather than considering eigenspaces of

$$\mathbf{R}W \xrightarrow{A=\pi^t \circ \pi} \mathbf{R}W$$

lets consider instead the eigenspaces of

$$\mathbf{R}^{\Phi_{\mathcal{O}}} \xrightarrow{B=\pi \circ \pi^t} \mathbf{R}^{\Phi_{\mathcal{O}}}.$$

General theory says they have the same **nonzero** eigenvalues, with eigenspaces carrying the same $W \times \mathbf{Z}/2\mathbf{Z}$-representations.

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Together with the representation theory, this already explains two of the four eigenspaces that we observed...

Decompose $\mathbf{R}^{\Phi_{\mathcal{O}}}$ as $\mathbf{Z}/2\mathbf{Z}$-module

$$\mathbf{R}^{\Phi_{\mathcal{O}}} = \left(\mathbf{R}^{\Phi_{\mathcal{O}}}\right)_+ \oplus \left(\mathbf{R}^{\Phi_{\mathcal{O}}}\right)_-$$

where

$$\left(\mathbf{R}^{\Phi_{\mathcal{O}}}\right)_+ \text{ has basis } \mathbf{R}\{e_\alpha + e_{-\alpha}\}_{\alpha \in \Phi_{\mathcal{O}} \cap \Phi_+}$$
$$\left(\mathbf{R}^{\Phi_{\mathcal{O}}}\right)_- \text{ has basis } \mathbf{R}\{e_\alpha - e_{-\alpha}\}_{\alpha \in \Phi_{\mathcal{O}} \cap \Phi_+}$$

The summand $\left(\mathbf{R}^{\Phi_{\mathcal{O}}}\right)_+$ carries the coset action of W on W/Z, where Z is the subgroup of W stabilizing the hyperplane H.

The easy calculation

$$B(e_\alpha + e_{-\alpha}) = \frac{|W|}{2} \sum_{\beta \in \Phi_{\mathcal{O}}} e_\beta$$

shows that $\left(\mathbf{R}^{\Phi_{\mathcal{O}}}\right)_+$

- lies almost entirely in the kernel (0-eigenspace) of B,
- except for containing a 1-dimensional $\frac{|\mathcal{O}||W|}{2}$-eigenspace.

The other summand $\left(\mathbf{R}^{\Phi_{\mathcal{O}}}\right)_{-}$, as W-representation carries the **twisted** coset action $\mathrm{Ind}_Z^W \chi$ where

$$\begin{array}{rcl} Z & \xrightarrow{\chi} & \{\pm 1\} \\ w & \longmapsto & w\big|_{H^\perp}. \end{array}$$

It would be nice if $\mathrm{Ind}_Z^W \chi$ were W-multiplicity-free,
so that we could apply that eigenvalue integrality principle...

Outline

What's a Gelfand pair?

A **Gelfand pair** (W, Z) is

- a group W
- and subgroup Z

such that the transitive action on the coset space $X = W/Z$ is **multiplicity-free** for W.

In other words, $\mathrm{Ind}_Z^W \mathbf{1}$ has no multiplicity in its W-irreducible decomposition.

What's a twisted Gelfand pair?

More generally, a **twisted Gelfand pair** (W, Z, χ) is

- a group W
- and subgroup Z
- and degree-one character $\chi : Z \to \mathbf{C}^{\times}$

such that $\mathrm{Ind}_Z^W \chi$ has no multiplicity in its W-irreducible decomposition.

Who can resist a juicy Gelfand pair?

Not this guy...

SOME q-KRAWTCHOUK POLYNOMIALS ON CHEVALLEY GROUPS

By DENNIS STANTON*

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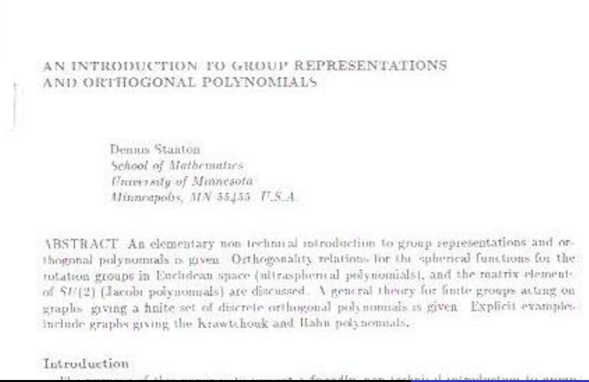
1. Introduction. The Krawtchouk polynomials are the eigenmatrices of the binary Hamming scheme, which is the set of all N-tuples of ± 1's. The automorphism group of this set consists of all sign changes and a permutation group on N entries. This group is the Weyl group of a simple Lie algebra. We can also describe the Krawtchouk polynomials as the spherical functions on the Weyl group modulo a maximal Weyl subgroup. Thus there is a natural set of q-Krawtchouk polynomials by replacing the

Need a Gelfand pair review...?

... and want it from the viewpoint of **orthogonal polynomials** and **hypergeometric functions**,
as **spherical functions** on W, or on $X = W/Z$?



AN INTRODUCTION TO GROUP REPRESENTATIONS AND ORTHOGONAL POLYNOMIALS

Dennis Stanton
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University of Minnesota
Minneapolis, MN 55455 U.S.A.

ABSTRACT An elementary non technical introduction to group representations and orthogonal polynomials is given. Orthogonality relations for the spherical functions for the rotation groups in Euclidean space (ultraspherical polynomials), and the matrix elements of $SU(2)$ (Jacobi polynomials) are discussed. A general theory for finite groups acting on graphs giving a finite set of discrete orthogonal polynomials is given. Explicit examples include graphs giving the Krawtchouk and Hahn polynomials.

Introduction

The twisted Hecke algebra

How to show $\mathrm{Ind}_Z^W \chi$ is W-multiplicity-free?

It's equivalent to show that its ring of W-endomorphisms, the **(twisted) Hecke algebra** inside $\mathbf{R}W$

$$\mathcal{H} := e_\chi \cdot \mathbf{R}W \cdot e_\chi$$

is **commutative.**

Here

$$e_\chi := \frac{1}{|Z|} \sum_{w \in Z} \chi(w^{-1}) w.$$

The twisted version of Gefland's trick

How to show $\mathcal{H}$ is commutative?

$\mathcal{H}$ is spanned by the **nonzero** elements $\{e_\chi w e_\chi\}$ obtained when one runs through the double cosets ZwZ in W.

PROPOSITION("twisted Gelfand's trick").
$\mathcal{H}$ is commutative if every double coset ZwZ with $e_\chi w e_\chi \neq 0$ contains an **involution** $w = w^{-1}$.

Proof.
These elements $e_\chi w e_\chi = e_\chi w^{-1} e_\chi$ are all fixed by the anti-automorphism $x \mapsto x^{-1}$ on $\mathbf{R}W$, and hence span a commutative subalgebra $\mathcal{H}$. $\square$

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The twisted Gelfand trick works for us

The double cosets ZwZ in our case (roughly) correspond to the **dihedral angles** $\angle\{H, H'\}$ between hyperplanes H, H' in the chosen W-orbit $\mathcal{O}$.

The cosets ZwZ giving $e_\chi w e_\chi = 0$ turn out to be those with H, H' **orthogonal.**

When the dihedral angle $\angle\{H, H'\}$ is not orthgonal **reduction to the dihedral case** shows that the coset ZwZ contains an involution.

This gives the first theorem: the eigenvalues of A lie in **Z**.

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And for the simply-laced theorem...

... one only needs double cosets ZwZ where $\angle\{H, H'\} \in \{0, \frac{\pi}{3}\}$.

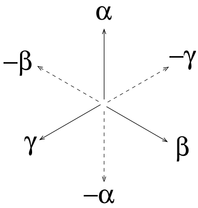
In this case, it turns out (stealing an idea from Renteln) that

$$\mathrm{Ind}_Z^W \chi \cong \mathbf{R}^\ell \oplus U$$

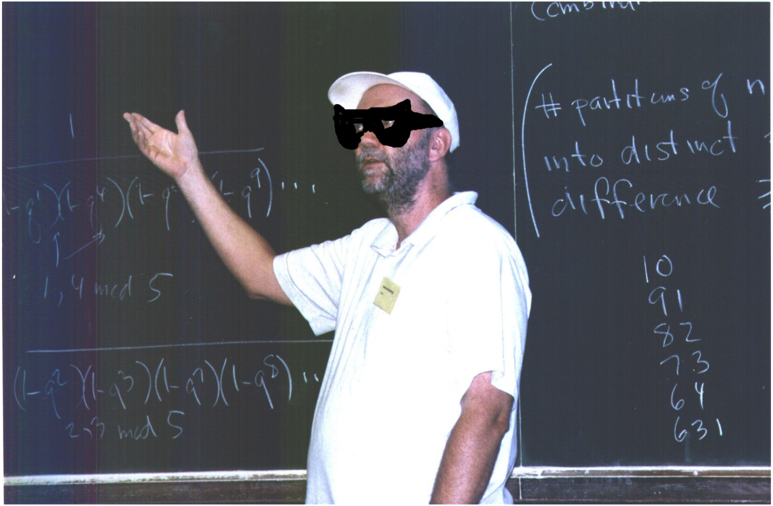
where U is a W-irreducible spanned by the vectors

$$\{e_\alpha + e_\beta + e_\gamma - (e_{-\alpha} + e_{-\beta} + e_{-\gamma})\}$$

running over α, β, γ as shown:



One mystery remains: Who **was** that masked man?



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