

Date due: September 19, 2005

Hand in only the five starred questions.

**Section 1.1** nos. 14, 25\*, 32, 34, 36.

**Section 1.2** nos. 4\*, 5, 7, 11.

- A. Let  $\alpha$  be a rotation about the origin in the plane and let  $\rho$  be the reflection in the  $x$ -axis. Show that  $\rho\alpha\rho^{-1} = \alpha^{-1}$ .
- B. \* Consider a pentagonal prism as shown, and let  $G$  be the group whose elements are the rotations of 3-dimensional space which leave the prism looking the same after doing the rotation as it looked before.
- (a) What is the order of  $G$ ?
  - (b) How many elements of order 2 does  $G$  have?
  - (c) Show that  $G$  is not commutative.

- C. Let  $n$  be a positive integer, and let  $G$  be the group whose elements are the  $2n$  symbols  $x^i y^j$  where  $i \in \{0, 1\}$  and  $j \in \{0, \dots, n-1\}$ , subject to the rules  $x^2 = y^n = e$  and  $xy = y^{-1}x$ . Express  $yx$  in the form  $x^i y^j$  with  $i \in \{0, 1\}$  and  $j \in \{0, \dots, n-1\}$ .

**Section 1.3** nos. 15, 19.

**Section 1.4** no. 11.

- D. \* Show that the group  $H(\mathbb{Z}/2\mathbb{Z})$  of 1.4 no. 11 is isomorphic to  $D_8$ .

**Section 1.5** no. 3\* (proving that you have indeed found generators and relations).