Math 8201 **Homework 7** **PJW**
Date due: October 31, 2005.

Hand in only the starred questions.
Section 4.3 2, 4, 5, 6*, 9, 10, 11, 13, 25, 29, 30, 31, 32, 34 (I list a lot of questions, and I expect that it will be appropriate for you to skim over many of them, simply looking to make sure you can do them.)

W. Let G be an infinite group containing an element $x \neq 1$ having only finitely many conjugates. Prove that G is not simple.

X. Let $a = (1, 2, 3, 4) \in S_4 = G$. Describe the centralizer $C_{S_4}(a)$. (Determine its structure and its order.)

Y. Show that when $a = (4, 5) \in S_5$, the subgroup $C_{S_5}(a)$ consists of $S_3 \cup S_3 a$, where S_3 denotes the symmetric group on three symbols as a subgroup of S_5 permuting the symbols $\{1, 2, 3\}$.

Z. (related to qn. 10) Consider the permutation $a = (1, 2, 3, 4, 5)$.
   (a) Show that a has 24 conjugates in S_5.
   (b) Show that a has only 12 conjugates in A_5. (Hint: compute the index of $C_{A_5}(a)$ in A_5.)
   (c) Show that $(1, 2, 3, 4, 5)$ is conjugate in A_5 to $(5, 4, 3, 2, 1)$.
   (d) Show that $(1, 2, 3, 4, 5)$ is not conjugate in A_5 to $(1, 3, 5, 2, 4)$.

AA. Let $a = (1, 2, 3, 4)(5, 6, 7) \in S_7$.
   (a) Find a permutation g of the symbols $\{1, 2, 3, 4, 5, 6, 7, 8\}$ so that

$$g(1, 2, 3, 4)(5, 6, 7)g^{-1} = (2, 1, 6, 5)(3, 8, 7).$$

   Express g as a product of disjoint cycles.
   (b) Calculate the number of conjugates of a in S_7. Calculate the number of conjugates of a in S_8.
   (c) Show that the only elements of S_7 which commute with a are the powers of a.

2*. (Graduate Algebra Exam, Fall 2002) (18%)
   (a) (4%) Calculate the numbers of conjugates of each of the elements $(1, 2, 3, 4, 5)$ and $(1, 2, 3)(4, 5, 6)$ in the symmetric group S_6.
   [We use cycle notation for permutations, writing them as a disjoint union of cycles.]
   (b) (7%) Calculate the numbers of conjugates of each of the elements $(1, 2, 3, 4, 5)$ and $(1, 2, 3)(4, 5, 6)$ in the alternating group A_6.
   (c) (7%) Show that $(1, 2, 3, 4, 5)$ and $(1, 3, 5, 2, 4)$ are not conjugate in A_6.

Section 4.4 4, 5*, 8, 11*, 12, 13*.