Math 8201 **Homework 11** **PJW**

Date due: Wednesday November 26, 2008. Hand in the 6 starred questions.

ZZ (a) How many essentially different ways are there to write $29 \cdot 37$ as a sum of square of two integers? We regard $a^2 + b^2 = b^2 + a^2 = (-a)^2 + b^2$ etc as 'the same'.
(b) How many essentially different ways are there to write $29 \cdot 31$ as a sum of square of two integers?
(c) How many incongruent right-angled triangles are there with hypotenuse of length $17^2 = 289$ and sides of integer lengths? (Only consider triangles with non-zero area.)

Section 3.8 3.82, 3.83

AAA* For every commutative ring R, prove that $R[x]/(x) \cong R$. Prove also that

$$R[x]/(x+1) \cong R[x]/(x-1).$$

For the moment we will only do Theorems 3.110-3.112 from section 3.8.

Section 6.1 6.1, 6.3, 6.5, 6.6*, 6.9, 6.11, 6.12*, 6.16*

BBB* Let R be a finite commutative ring with identity. Prove that every prime ideal of R is a maximal ideal.

CCC* Let R be a (not necessarily commutative) ring whose only left ideals of (0) and R. show that R is a division ring.

DDD Let I and J be ideals of a commutative ring R and assume P is a prime ideal of R that contains $I \cap J$. Prove that either I or J is contained in P.

EEE Let R be a commutative ring and suppose for each $a \in R$ there is a positive integer n (depending on a) such that $a^n = a$. Prove that every prime ideal of R is a maximal ideal.