Math 8201 **Homework 2** **PJW**
Date due: September 22, 2008

There will be a 30 minute quiz in class on this date on the subject matter of Homeworks 1 and 2. You may be asked about the topic of any of the questions on these homework sheets, including the questions you did not hand in. The quiz questions will be like (and perhaps easier than) the questions on the homework sheets. If you can do the homework questions, you can do the quiz questions.

Hand in only the 6 starred questions.

Section 2.4

H Let H_1 and H_2 be subgroups of G. Show that any right coset relative to $H_1 \cap H_2$ is the intersection of a right coset of H_1 with a right coset of H_2 Use this to prove *Poincaré's Theorem* that if H_1 and H_2 have finite index in G then so has $H_1 \cap H_2$.

I Find the inverse of $\overline{17}$ in $U(\mathbb{I}_{49})$.

J Show that $U(\mathbb{I}_{13})$ is a cyclic group.

Section 2.5 nos. 2.41, 2.43, 2.44, 2.46*, 2.47, 2.56, 2.58, 2.61, 2.62*, 2.64*

Section 2.6 nos. 2.67, 2.68*, 2.69, 2.70, 2.71, 2.74*, 2.76*

K Let N be a subgroup of index 2 of a finite group G. Show that all the elements of G which do not lie in N have even order. [You may want to use, among other things, Exercise 2.47(i).]

L Show that if A is a subgroup of G of index 2 then for any subgroup H of G, $|H : H \cap A|$ equals 1 or 2.