Math 8201 **Homework 4** **PJW**

Date due: October 6, 2008 Hand in the 5 starred questions.

There will be a 30 minute quiz in class on this date on the subject matter of Homeworks 3 and 4. You may be asked about the topic of any of the questions on these homework sheets, including the questions you did not hand in. The quiz questions will be like (and perhaps easier than) the questions on the homework sheets. If you can do the homework questions, you can do the quiz questions. As in Quiz 1 you may assume any results you need from the book, class or homework, and may consult these, provide that this does not invalidate the question.

Section 2.7 nos. 2.78, 2.82, 2.85(ii)*, 2.88, 2.94*, 2.95*, 2.96, 2.97.

R Let $a = (1, 2, 3, 4) \in S_4 = G$. Describe the centralizer $C_{S_4}(a)$ and the normalizer $N_{S_4}(\langle a \rangle)$. (Determine their structure and order.)

S Show that when $a = (4, 5) \in S_5$ we have $C_{S_5}(a) = S_{\{1,2,3\}} \times S_{\{4,5\}}$.

T Compute $C_G(a)$ and $N_G(\langle a \rangle)$ for all elements a in G when G is dihedral of order 10 and of order 12. Compare the answers with the number of conjugates of the element and whether the subgroup is normal.

U* (Compare with exercise 2.89.) Consider the permutation $a = (1, 2, 3, 4, 5)$.

- (a) Show that a has 24 conjugates in S_5.
- (b) Show that a has only 12 conjugates in A_5. (Hint: compute the index of $C_{A_5}(a)$ in A_5.)
- (c) Show that $(1, 2, 3, 4, 5)$ is conjugate in A_5 to $(5, 4, 3, 2, 1)$.
- (d) Show that $(1, 2, 3, 4, 5)$ is not conjugate in A_5 to $(1, 3, 5, 2, 4)$.

V Let $a = (1, 2, 3, 4)(5, 6, 7) \in S_7$. Show that the only elements of S_7 which commute with a are the powers of a. What elements commute with a when it is regarded as an element of S_8?

W* Let G be an infinite group containing an element $x \neq 1$ having only finitely many conjugates. Prove that G is not simple.

X If G is a group of odd order, prove for any nonidentity element $x \in G$ that x and x^{-1} are not conjugate in G.