Math 8201 **Homework 5** **PJW**

Date due: October 13, 2008 Hand in the 5 starred questions.

Section 2.7 again. For some of these questions use the results of exercises 2.69 and 2.82 which have been listed on earlier homework sets.

Y Show that every subgroup of order 4 of the dihedral group D_{12} of order 12 contains the center $Z(D_{12})$. [You may assume that $Z(D_{12}) = \langle\sigma\rangle$, where σ is rotation through 180°.]

Z* Let G be a simple group of order $168 = 8 \cdot 3 \cdot 7$. Show that G has no proper subgroup of size larger than 24.

AA* Let P be a group of order p^3 for some prime p. Prove that x^p lies in the center $Z(P)$, for every element x of P.

BB* Prove that if p is a prime and P is a subgroup of S_p of order p, then $N_{S_p}(P)/C_{S_p}(P) \cong \operatorname{Aut} P$.

CC Let G be a group of order 3825. Prove that if H is a normal subgroup of order 17 in G then $H \leq Z(G)$.

DD* Let G be a group of order 203. Prove that if H is a normal subgroup of order 7 in G then $H \leq Z(G)$. Deduce that G is abelian in this case.

EE* Let Q_8 be the quaternion group of order 8.

(a) Prove that Q_8 is isomorphic to a subgroup of S_8.

(b) Prove that Q_8 is not isomorphic to a subgroup of S_n for any $n \leq 7$.

FF (3 pts) Prove that the dihedral group D_{32} is not isomorphic to $D_{16} \times Z_2$.

GG Let $D_8 = \langle\rho, \sigma\rangle$ where ρ is rotation through $\pi/2$. Calculate the cycle type of ρ as a permutation of D_8 in the left regular action of D_8. [The cycle type of a permutation is a list of the lengths of the cycles which arise when the permutation is expressed as a product of disjoint cycles.]

Section 5.2 (I have not put stars by any of the following questions because I will probably want to set them next week.) 5.18, 5.19, 5.28, 5.29, 5.30, 5.31

HH (This is more general than exercise 5.25.) Show that $O_p(G)$ equals the intersection of the Sylow p-subgroups of G.