Math 8201 **Homework 8** **PJW**

Date due: November 3, 2008. There will be a quiz on this date. Hand in the 6 starred questions.

Section 5.5 The material of this section is not explicitly on the syllabus, and I do not recall ever seeing a question on presentations on the graduate algebra exam. On the other hand you are likely to be confronted with presentations of groups the moment you start doing group theory outside this course. You should be able to handle the results of 5.80 (in the case of the group of order 8), 5.81, 5.82 and 5.83. You should have a rough idea of what a presentation means. For the purposes of this course you do not need to know about free groups and the official definition of a presentation given on page 306.

PP Find the orders of the groups

(i) $\langle x, y \mid x^{20}y^{21} = 1 = x^{21}y^{22}\rangle$

(ii)* $\langle x, y \mid x^8 = y^2 = 1, yxy = x^2\rangle$

(iii) $\langle x, y \mid x^8 = y^2 = 1, yxy = x^4\rangle$

QQ Show that $\langle x, y \mid x^2 = y^2 = 1 = (xy)^n\rangle$ is a presentation for a dihedral group.

RR* Show that $\langle x, y \mid y^3 = 1, yxy^{-1} = x^2\rangle$ is a presentation for the group of 2×2 matrices with entries in $\mathbb{I}_7$ generated by $\begin{pmatrix} \bar{2} & \bar{0} \\ \bar{0} & \bar{1} \end{pmatrix}$ and $\begin{pmatrix} \bar{1} & \bar{1} \\ \bar{0} & \bar{1} \end{pmatrix}$.

SS Find a presentation for the alternating group A_4, showing that you have indeed given a presentation for this group.

5.66 on page 311.

Section 3.2 3.10*, 3.12, 3.14, 3.15, 3.16, 3.17*, 3.18

Section 8.1 8.1, 8.8, 8.10*, 8.11, 8.21

TT* Find a ring R and elements a, b, c all distinct from 0 such that $a\cdot b = a\cdot c$ and yet $b \neq c$.

UU Show that the quaternions z for which $z^2 + 1 = 0$ are precisely those which may be written $z = bi + cj + dk$ with $b^2 + c^2 + d^2 = 1$.
[Hint: you may want to show as a first step that if z satisfies the equation then $z = \pm\bar{z}$, and then go on to show that in fact $z = -\bar{z}$. Now continue.]