3593 Exam 2 practice questions
Sections of the book to be tested: 3.5, 3.6, 3.7, 4.1, 4.3, 4.5 and 4.8. You don't need to know the Cayley-Hamilton theorem 4.8.25. In this exam you will not be tested on eigenvalues and eigenvectors. The emphasis of the exam is on using the theory in actual situations, in spite of the fact that the first question below asks you to prove something. I will not ask you for a proof in the exam. It is just that question 1 below is good practice.

1. Prove that a subset of a set of volume zero has volume zero.

2. Consider the functions defined on $\mathbb{R}^2$

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{cases} 1 & \text{if } y = x^2 \text{ and } -1 \le x \le 1 \text{ is rational,} \\ 0 & \text{otherwise} \end{cases},$$

$$g\begin{pmatrix} x \\ y \end{pmatrix} = \begin{cases} xy & \text{if both } x \text{ and } y \text{ are rational between } -1 \text{ and } 1, \\ 0 & \text{otherwise.} \end{cases}$$

Do $\int_{\mathbb{R}^2} f \, |d^2x|$ and $\int_{\mathbb{R}^2} g \, |d^2x|$ exist? If so, what are their values?

3. Let σ be the permutation $\sigma(1) = 2, \sigma(2) = 3, \sigma(3) = 4, \sigma(4) = 1$ and let τ be the permutation $\tau(1) = 1, \tau(2) = 4, \tau(3) = 3, \tau(4) = 2$. What is the sign of the permutation $\sigma\tau$?

Relevant questions from the book:
Section 3.5 page 342: I list again the questions from HW 4: 1, 2, 3, 3b*, 3c*, 4, 4b*, 5, 8, 9, 10, 10d*, 11, 17, 17c*, 17d*
Section 3.6 page 350: the questions from HW 4 were: 1, 2, 6, 7, 8.
Section 3.7 page 368: questions from HW 5 were: 1, 2, 3*, 4*, 5, 6, 7, 8*, 9, 10a*, 10b*, 11, 13, 14 plus an extra questions which might be worth looking at. Also numbers 20, 21, 22 from Section 3.9 below are relevant.
Section 3.9 page 386: 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 25
Section 4.1 page 406: 10, 14, 15
Section 4.3 page 429: 5
Section 4.5 page 447: 7, 8, 11, 12, 14, 15, 16, 18
Section 4.8 page 476: 2, 12, 13, 15
Section 4.12 page 516: 11, 12, 13