

For each of the following, indicate whether or not you 'know' it. Let p be a prime and G a finite non-identity p -group.

1. $Z(G) \neq 1$.
2. If $G/Z(G)$ is cyclic then $G = Z(G)$.
3. The center of a non-abelian p -group of order p^3 has order p .
4. The lower and upper central series of G have the same length.
5. If $H < G$ then $H < N_G(H)$.
6. If H is a maximal subgroup of G then H is normal in G .
7. Every non-identity normal subgroup of G contains a non-identity element of $Z(G)$.
8. $G/\Phi(G)$ is the maximal elementary abelian quotient of G .
9. Each set of elements of G which generates G contains a generating set of size $d(G)$, where $d(G)$ is the minimum size of a set of elements which generates G .
10. The number of maximal subgroup of G is $(p^{d(G)} - 1)/(p - 1)$.