

Date due: April 9, 2018. There will be a quiz on this date.

Hand in only the starred questions.

Section 14.1, page 546 5, 6\*, 7, 8\*, 9\*, 10.

Section 14.2, page 562 3, 4\*, 5\*, 6.

J. (Fall 2001, qn. 5) (15%) Let  $f(x) = x^4 + ax^3 + bx^2 + ax + 1 \in \mathbb{Q}[x]$ , and let  $E$  be the splitting field of  $f$ .

(a) (3) Show that for each root  $\alpha$  of  $f$  in  $E$ , also  $\alpha^{-1}$  is a root of  $f$ .

(b) (5) Show that  $[E : \mathbb{Q}] \leq 8$ .

(c) (7) Show that if  $f$  is irreducible in  $\mathbb{Q}[x]$  and has exactly two real roots and two complex roots, then the Galois group  $G_{E/\mathbb{Q}}$  is isomorphic to  $D_4$ , the dihedral group of order 8.

K. (Spring 1995, qn. 3) (15%) Let  $f(X) \in k[X]$  be a polynomial of degree  $n$  over  $k$  and let  $K$  be a splitting field for  $f$  over  $k$ . Suppose that the Galois group  $\text{Gal}(K/k)$  is the symmetric group  $S_n$ .

(a) (7) Show that  $f$  is irreducible in  $k[X]$ , and separable.

(b) (5) Let  $\alpha$  be a root of  $f$  in  $K$ . Show that in  $k(\alpha)[X]$ ,  $f$  factorizes as

$$f(X) = (X - \alpha)g(X)$$

where  $g(X)$  is an *irreducible* polynomial.

(c) (3) Determine the Galois group of  $g$  over  $k(\alpha)$ .