Math 8202 **Homeworks 5 and 6** **PJW**

Date due: March 5, 2018. There will be a quiz on this date.

Hand in only the starred questions. We were behind in covering the material for Homework 5 as first announced, so Homeworks 5 and 6 are combined into one, with some questions removed.

Section 12.2 nos. 1, 3*, 5, 6, 9, 10*, 11, 12, 15* 18*, 19. (Also questions 13, 14, 16, 17 are good.)
Section 12.3 nos. 1*, 2, 7b, 9*, 16, 18*, 24*, 32, 33, 34. All of questions 4 – 39 are good (omitting the parts of questions such as in question 11 which have to do with the rational canonical form).

The algorithm presented in the book at the end of Section 12.3 for putting a matrix into Jordan canonical form (and finding a basis which achieves this) first puts the matrix into rational canonical form. This probably works quite well and fits with our work on Smith normal form, but it is not the algorithm I would recommend, which is technically more elementary. The algorithm I will teach to find an appropriate basis for the generalized eigenspace for an eigenvalue λ of a matrix A first finds the smallest n such that $(A-\lambda I)^n = 0$. Now find a basis for the whole space modulo the nullspace of $(A-\lambda I)^{n-1}$. Apply $(A-\lambda I)$ to these elements and extend them to a basis of the nullspace of $(A-\lambda I)^{n-1}$ modulo the nullspace of nullspace of $(A-\lambda I)^{n-2}$. Apply $(A-\lambda I)$ to these elements and extend them to a basis of the nullspace of $(A-\lambda I)^{n-2}$ modulo the nullspace of nullspace of $(A-\lambda I)^{n-3}$. And so on. Eventually we will have a basis of the generalized eigenspace with respect to which A is in Jordan canonical form.

It is already laborious to do this with a 3×3-matrix, and about the most that could be done in exam conditions is a straightforward 4×4-matrix, or a matrix which has a special structure. Having said that, putting a matrix into Jordan canonical form is a routine thing, and you should get practice so that you can do it automatically without thinking too much.