Equations of lines and planes

We are familiar with various forms for the equation of a line in $\mathbb{R}^2$. There is the slope–intercept form

$$y = mx + c$$

where m is the slope and c is the intercept with the y-axis. This works as long as the line is not parallel to the y-axis, in which case we should use some other form of the equation such as

$$ax + by = c$$

or possibly

$$\frac{x-u}{s} = \frac{y-v}{t}$$

which gives a line passing through (u, v). This has as a special case the point-slope form $y - v = m(x - u)$. We can also describe the line parametrically, as the set of points $\underline{u} + t\underline{v}$ where t is allowed to vary through $\mathbb{R}$, for a line passing through $\underline{u}$, in the direction of $\underline{v}$. To be able to convert between this forms of the equations is important.

Problems: 1. Convert the line $2x + 5y = 7$ in $\mathbb{R}^2$ into: (a) slope-intercept form, (b) at least one point-slope form, (c) at least one parametric form.

2. Find the intersection of $2x + 5y = 7$ with (a) $4x + 10y = 7$, (b) $4x + 10y = 14$, (c) $x + 2y = 1$.

In $\mathbb{R}^3$ a line needs two equations to define it, such as

$$\frac{x_1 - u_1}{w_1} = \frac{x_2 - u_2}{w_2} = \frac{x_3 - u_3}{w_3}$$

for a line passing through the point (u_1, u_2, u_3). We can also describe a line parametrically as $\underline{u} + t\underline{v}$ where t is allowed to vary through $\mathbb{R}$, for a line passing through $\underline{u}$, in the direction of $\underline{v}$ (the same as with $\mathbb{R}^2$, except now the vectors are in $\mathbb{R}^3$). It is important to be able to convert between these descriptions of a line, and we should be able to find some point lying on a line, and also a vector pointing in the direction of the line, no matter how the line is given.

Problems: 1. Express the line given parametrically as $(1, 1, 1) + t(2, 3, 5)$ in the form above where there are two equations.

2. Find a parametric form for the line

$$x - 1 = y + 2 = \frac{z-3}{2}.$$

3. To do 2. you will have solved the problems: find a point on the line given in 2; find a vector pointing in the direction of the line given in 2. These are legitimate questions in their own right.