Math 5990 Topics in Mathematics: Characters of Finite Groups
Fall 2018

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Groups are a way to study symmetry in abstract, so that whenever we have an object which has some aspect of symmetry to it there is an associated group that is a measure of the amount of symmetry and the structure that the symmetry has. If the symmetry happens to be geometrical in nature, taking the form of linear transformations of space, then the group may be realized as a group of invertible matrices. The study of the ways of realizing a group by means of invertible matrices is representation theory. Apart from having inherent interest due to the appeal of symmetry, the theory has applications in physics and chemistry, wherever there is symmetry.

We will start with a brief introduction to group theory, and at this point of the course there will be overlap with other abstract algebra courses. After that we will focus on group representations over the real and complex numbers, and their associated characters. Applications discussed will include the vibration of molecules and crystallography.

Text for the course: G. James and M. Liebeck, Representations and Characters of Groups, 2nd edition, Cambridge U.P. 2001, (paperback, about $70)

Prerequisites: Linear algebra, including properties of bases such as the facts that any spanning set of vectors contains a basis, every independent set of vectors can be extended to a basis. I will assume you know that given a basis, a linear map may be represented by a matrix, and that you are able to compute such a matrix of a linear map (the columns of the matrix are the images of the basis vectors). The other main prerequisite I will assume is the mathematical sophistication to handle proofs of results and written arguments. Knowledge of group theory is not a prerequisite.