Math 3593H Honors Mathematics II Spring Semester 2011

Assignment 12 - Due Thursday 4/21/2011

Read: Hubbard and Hubbard Sections 6.5, 6.6 and 6.7.

Exercises:
Section 6.5 (pages 604-606): 1, 2*, 3, 4, 5, 6*, 7*, 8, 9*, 11, 12, 15, 16, 17, 18*, 20*.
I think question 11 should read: 'Evaluate the work form of each ...'
Section 6.6 (pages 618-620): 3, 5a*, 5c*, 5d*, 5e*, 8.
Section 6.7 (pages 625-626): 1 - 11, 2*, 8*, 9*.

Comments:
Section 6.5 makes the connection between the language of forms which we have been using, and the type of integrals that you will probably see in physics, or that you would see in a different mutlivariable calculus class.

Section 6.6 is a complicated treatment of boundaries, of a degree of complication which we do not need in order to understand Stokes' theorem. Unfortunately we do need to know what a manifold with boundary is, because these are essential for the theorem, so we cannot skip Section 6.6. Aside from knowing what a manifold with boundary is, the other thing we need to know is that an orientation of the manifold determines an orientation of the boundary, obtained by standing on the boundary on the side away from the rest of the manifold, and requiring that a basis of the tangent space to the boundary is direct if the basis for the tangent space to the manifold obtained by inserting a vector pointing away from the manifold before the basis for the tangent space to the boundary is direct. The definition of a manifold with boundary we will use is that it is a subset of n-dimensional space which locally can either be parametrized in the usual way, or else locally is the restriction of a usual parametrization to a half space in which the first coordinate is non-negative. The boundary of a k-dimensional manifold is then a (k-1)-dimensional manifold, and we recognize things like a half-space, a solid ball and a solid torus as examples of this.

I feel rather safe in advising you not to read Section 6.6 at all. For the homework exercises I have excluded parts of questions which require you to know what they mean by a 'piece with boundary'.

I propose not to define the exterior derivative in the way they do in Definition 6.7.1. Instead I think it is more straightforward to define it by means of its algebraic properties, most of which are listed in Theorem 6.7.2. The other two algebraic properties (which we deduce from 6.7.2) are given in 6.7.7 and 6.7.8. The statement of 6.7.1 will then become a theorem, instead of a definition. Almost all the homework exercises you are asked to do are done by applying these algebraic properties.

I think it is not worth troubling too much about the statement of 6.7.1. It is true that I have asked you to do some homework verifying the property of 6.7.1, but perhaps we should not try to understand it too much. When we discuss 6.7.1 I will approach it in the special case of an elementary form and a parallelogram aligned with the coordinate axes. This makes life easier, and the general case is deducible from this.

Something like 6.7.1 is needed for the informal proof of Stokes' theorem which we will do in section 6.9. They do not actually prove Stokes' theorem in that section, and this is a good choice the authors have made, but they do give some idea of why it is true, and we will also discuss this. However, the informal statement about the relationship between the exterior derivative and the boundary of a manifold which I would prefer to use is a little different to 6.7.1. This is why I think we should not trouble ourselves too much with it.