Theory check.

1. Every subgroup S in G of index 2 is normal.
2. Let $H \triangleleft G$ have index n. If $y \in G$, prove that $y^n \in H$. Give an example to show this may be false when H is not normal.
3. Let $H \triangleleft G$ and let $\pi : G \to G/H$ be the natural map. Assume X is a subset of G for which $\pi(X)$ generates G/H. Prove that $G = \langle H \cup X \rangle$.
4. Let $N \triangleleft G$ and let $f : G \to H$ be a homomorphism whose kernel contains N. Then f induces a homomorphism $f_* : G/N \to H$, namely $f_*(Na) = f(a)$.
5. The commutator subgroup G' is a normal subgroup of G.
6. If H is any normal subgroup of G then G/H is abelian if and only if $G' \subseteq H$.
7. Let H be a subgroup of G that contains G'. Prove that $H \triangleleft G$.
8. Show that it is possible to have $G = H \times K = H \times L$ (internal direct products) with $K \neq L$.
9. Find all subgroups of A_4. Using a Hasse diagram, display the subgroup lattice.

Example check: how many groups can you think of ?

In 5 minutes make a list of all the non-abelian groups you can think of which have order at most 500. For each group you name write down one (interesting) fact. Your score for the list will be the sum of the group orders.